MOTHER'S TIME ALLOCATION, CHILD CARE AND CHILD COGNITIVE DEVELOPMENT

Ylenia Brilli*†

Abstract. This paper analyzes the effects of maternal and non-parental time on a child's cognitive development. Using data from the Panel Study of Income Dynamics, we estimate a model that allows the mother's time productivity to depend on her education level, and that distinguishes between formal and informal care. The results show that child-care time of high-educated mothers is more productive than that of low-educated mothers and that of non-parental care. The simulation of policies subsidizing mothers' wages or regulating the non-parental care market indicates that children with low-educated mothers benefit more from replacing maternal time with non-parental time.

JEL classification: D13, J13, J22, C15.

Keywords: mother employment; mother time allocation; non-parental child care; child development; structural estimation.

*Department of Economics, Ca' Foscari University of Venice; Department of Economics, University of Gothenburg. E-mail: ylenia.brilli@unive.it

†I am indebted to Daniela Del Boca for her constant encouragement and advice. I am also grateful to Jérôme Adda, Cristian Bartolucci, Raquel Bernal, Gordon Dahl, Christopher Flinn, Marco Francesconi, James Heckman, Andrea Ichino, Chiara Monfardini, Stefano Mosso, Marco Tonello, Frank Vella, Matthew Wiswall, and participants at several seminars and conferences. The comments of the editors Isaac Ehrlich and John Eric Humphries and of two reviewers substantially improved the paper. This research was initially conducted at New York University and at Collegio Carlo Alberto; the hospitality of both institutions is gratefully acknowledged. The computations were performed using resources at Chalmers Centre for Computational Science and Engineering (C3SE), provided by the Swedish National Infrastructure for Computing (SNIC), and at High Performance Computing (HPC) implemented by the Venice centre in Economic and Risk Analytics for public policies (VERA). I gratefully acknowledge the financial support of Collegio Carlo Alberto and the European Union's Seventh Framework Programme under grant agreement no. 320116 for the research project FamiliesAndSocieties. Any errors should be attributed to the author.

1. Introduction

There has been long-standing interest in the social sciences in learning about the production of child cognitive achievement. As maternal child-care time is considered one of the most valuable inputs for child development (Cunha et al. 2006), the increase in the maternal employment rate and the associated rise in the use of non-parental forms of child care have raised concerns about the impact they might have on children. While there is an extensive literature on the effects of maternal employment on children's development,[1] clear evidence regarding the role played by the quality of the time the child spends with the mother and the alternative forms of care is still lacking.

This paper analyzes the effects of maternal employment and non-parental child care on children's cognitive development by taking into account the mothers' time allocation choice between child care and leisure and the potential heterogeneity in the productivity of maternal and non-parental child care.

In the US, the participation of mothers in the labor market increased from around 50% in the 1970s to more than 70% by the end of the 1990s (US Census Bureau 2000). Recent data from the American Time Use Survey (ATUS) indicates that working and non-working mothers may allocate their time out of work differently, as working mothers may want to prioritize the time spent with their children over leisure.[2] While an important determinant of the maternal labor supply is the level of education of mothers, Guryan et al. (2008) document that higher-educated mothers spend more time caring for their children than do the lower-educated mothers despite facing a higher opportunity cost for spending time in activities other than work. This suggests that a mother's education not only affects her labor market participation but may also affect her time allocation between child care and leisure, and, ultimately, her child's human capital. Furthermore, several studies have documented a striking increase in the use of non-parental child-care services during the last few decades for both working and non-working mothers (Bianchi 2000; US Census Bureau 2012). This may suggest that non-parental child care can be used, from the mother's viewpoint, not only for custodial purposes in the case that the

[1] See Ermisch and Francesconi (2005) for a review.

[2] By considering ATUS data for the period 2005-2009, US Census Bureau (2013) reports that employed mothers work, on average, five hours per day, and spend with their children only 30 minutes less than their non-employed counterparts. In contrast, employed mothers spend, on average, 2.5 hours per day in activities such as socializing, doing sports, or watching TV against the four hours per day spent by non-employed mothers.

mother works and needs someone to look after the child but also for educational purposes, especially before the child begins formal schooling.

In this paper, we estimate a model in which maternal labor supply and time allocation, as well as the use of non-parental child care, are considered to be endogenous choices of the mother, while the child cognitive development depends on the amount of time the child spends with the mother and on the amount of time the child spends in formal or informal care.[3] The model allows us to estimate the elasticity of a child's cognitive ability with respect to maternal child care and to the different types of non-parental care. Furthermore, the elasticity of a child's cognitive ability with respect to maternal care is allowed to vary depending on the mother's level of education. More precisely, we distinguish between mothers with more than 12 years of education (*high educated*) and mothers with 12 years of education or less (*low educated*). The estimation of such a model makes it possible to deal with the endogeneity and the simultaneity of the mothers' choices and to identify the contributions of both maternal child-care time and non-parental child care in the cognitive development of the child.

Several studies have assessed the effects of maternal employment or non-parental child-care use on the cognitive development of children, but only a few papers have evaluated the impact of these choices simultaneously using a structural approach. One group of studies estimates discrete-choice models in which the mother makes employment and non-parental child-care decisions. Bernal (2008) finds that one year of maternal employment and non-parental child care has a substantial negative effect on a child's cognitive ability, as it reduces a child's test scores by 1.8%.[4] However, Griffen (2019) and Rodríguez (2021) focus on high-quality formal child care and show that expanding the use of this service has large positive effects on children's cognitive skills.[5]

[3]While the informal services refer to the care provided by relatives, friends, or babysitters, the formal sector of the non-parental child care market includes center-based programs (such as daycare centers, nurseries, preschools, and after-school programs) and family daycare facilities. This distinction between formal and informal arrangements will be used throughout the whole paper.

[4]Other studies find negative effects of maternal employment only (Ermisch and Francesconi 2013; Mroz et al. 2010), while Bernal and Keane (2011) show that the negative effect of non-parental child care mainly stems from the use of informal child-care services. Agostinelli and Sorrenti (2018) argue that the negative effect of maternal employment is mainly due to a nonpositive substitution effect induced by the fact that the alternative form of care is of lower quality than the mother's child-care time.

[5]Berlinsky et al. (2020) estimate a model that endogenizes both demand and supply of non-parental child care and show that a combination of quality regulation and vouchers for working families leads to the greatest gains in child development.

While these papers account for the simultaneity of the employment and non-parental child care decisions, they do not consider the additional choice that a mother may make regarding her time allocation between time with her child and leisure. Thanks to the recent availability of data on direct measures of parental time investments, a second group of studies models mothers' decision-making process by accounting for their additional choice between child care and leisure. Del Boca et al. (2014) consider both parents' time investments in their child's ability and find that the productivity of a mother's time declines with a child's age, while a father's time with the child becomes more productive than the mother's when the child reaches adolescence. Different from Del Boca et al. (2014), this paper focuses on an earlier developmental stage of the child when the main substitute for a mother's child-care time is non-parental child care and accounts not only for differences in productivity between formal and informal child-care services but also for the heterogeneity in mothers' child-care time productivity induced by a mother's level of education. Similarly, Mullins (2020), Caucutt et al. (2020), and Moschini (2021) account for the multidimensionality of parental investment decisions and consider a child's human capital technology with maternal time and non-parental child care as inputs.

This paper contributes to the existing literature on the effects of parental decisions on children's cognitive development as follows. By allowing the child development production function to depend on both maternal child-care time and non-parental child care, the model accounts for the fact that mothers may choose not only how many hours to work and to use non-parental child care, but also how much time to devote to their child instead of engaging in leisure activities. This improves over the previous papers estimating discrete-choice models, that assume a one-to-one relationship between a mother's time out of work and a mother's child-care time. Furthermore, compared with other studies that use direct measures of maternal time investments (such as Del Boca et al. (2014), Caucutt et al. (2020) and Moschini (2021)), we provide novel evidence that maternal investments on the child's human capital depend not only on the productivity of a mother's child-care time but also on the productivity of the alternative forms of care.

In the model estimated in this paper, the mother's utility maximization problem is subject to a time and a budget constraints, as well as to the child's cognitive ability production function. The mother cares about consumption, leisure, and the child's cognitive ability. The child cognitive ability depends on the mother's child-care time, which may have heterogeneous impacts depending on the mother's level of education, and on the amount of time the child spends in formal or informal care. In each period, the mother decides her own labor supply and the investments in the child-development process. The empirical specification of the model takes into account that mothers who work and use non-parental child care are systematically different from those who do not. The model allows mothers to allocate their time between labor, time with the child, and leisure, depending on their preferences, their productivity in the labor market, and their productivity in the child-development process.

The model is estimated using US data from the Panel Study of Income Dynamics (PSID) linked to data from the Child Development Supplement (CDS) and the Time Diary (TD) component. The CDS provides information on all child-care arrangements used from birth until kindergarten and on the arrangement currently used at the time of the survey if the child is beyond kindergarten age. At every point in time, it is possible to observe what type(s) of child care arrangement the mother is using for the child (whether formal, informal, or both), the weekly amount of time each arrangement is used, and the hourly price paid for each arrangement. The TD component provides unique information on the amount of time the child spends with the mother, while the main PSID surveys give detailed information on the mother's work history and household income during the child's life cycle. The parameters of the model are retrieved using a Method of Simulated Moments estimator, which minimizes the distance between several data statistics and their model counterparts.

The results show a strong heterogeneity in the elasticity of a child's ability with respect to a mother's child-care time according to the mother's level of education, as the effect of maternal child care for high-educated mothers is almost double that of low-educated mothers. The elasticity of a child's ability with respect to non-parental child care also differs according to whether the service is formal or informal; formal child care is found to be more productive than informal child care, especially during a child's first years of life. Overall, however, a mother's child-care time is more productive than any type of non-parental child care, regardless of her level of education. This implies that an increase in a mother's labor supply induces a reduction in a child's ability through a decrease in the mother's child-care time, which may not be compensated for by the increase in non-parental child-care use.

The estimated model is used to simulate the effects of policies aimed at increasing the maternal labor supply or at regulating the non-parental child care market. The results confirm that the effects of the policies on mothers' labor supply and investment decisions, and children's cognitive outcomes are affected by the mother's level of education. In fact, high-educated mothers, who are more productive than low-educated mothers in the child cognitive development process, are less willing to reduce their child-care time in favor of non-parental child care. Concerning the effects on children's cognitive development, policies increasing mothers' labor supply have a nonpositive effect on the test scores of children with high-educated mothers, who lose more by replacing their time with the alternative care provider's time. Conversely, policies regulating the non-parental child care market in such a way that only high-quality arrangements are available have a larger positive effect on the test scores of children with low-educated mothers than on the test scores of children with high-educated mothers.

The remainder of the paper is organized as follows. Section 2 presents key stylized facts regarding maternal time allocation and non-parental child care use in the PSID-CDS data used for the model estimation. Section 3 describes the model that is estimated, and Section 4 introduces the data. Section 5 discusses the empirical strategy used for the identification of parameters, while Section 6 presents the results and the fit of the model. Section 7 reports the results of the policy simulations, and Section 8 concludes.

2. Background

This section reports key stylized facts about the time allocation of mothers and the use of formal and informal child care in the US, that motivate the model specification presented in Section 3. The evidence presented in this section is derived from the PSID-CDS data used for the model estimation, that provide information on the amount of time children spend with their mother and in non-parental child care.[6]

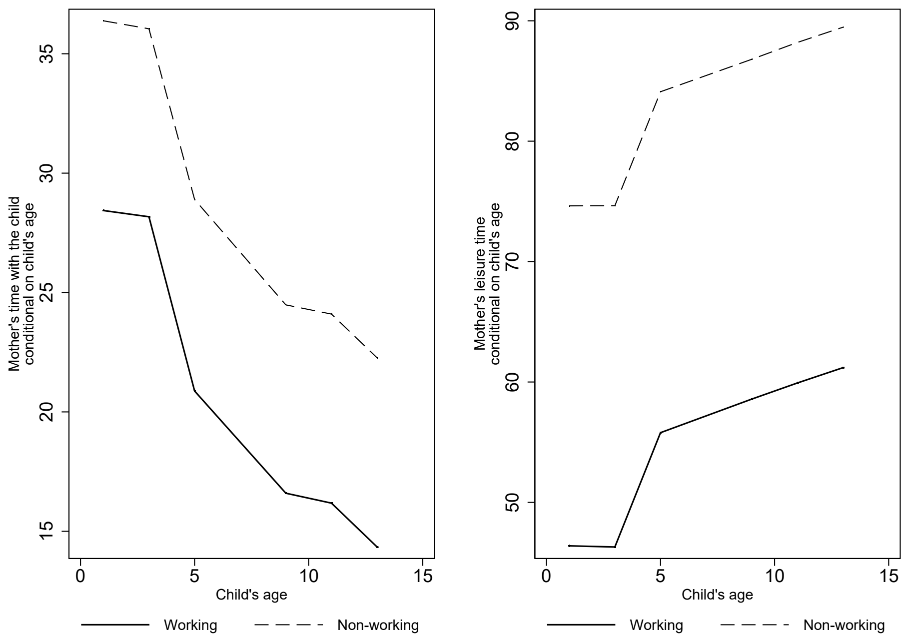
Figure 1 plots the fitted values from two regressions where the dependent variables are maternal child-care time and leisure time, regressed on a child's age fixed effects and a binary variable indicating whether the mother works in each period.[7] The graph on the left shows that the maternal child-care time of employed mothers is lower than that of the

[6]See Section 4 and Appendix B for a description of the dataset.
[7]Leisure time is computed as the difference between the total time endowment, assumed to be 112 hours per week, and the sum between working time and time with the child.

non-employed ones. However, the graph on the right indicates that employed mothers spend a lower amount of time out of work in leisure, while the corresponding level for non-working mothers is considerably higher. Note that while the difference in maternal time with the child between working and non-working mothers is equal to 8 hours per week, the difference in leisure time is equal to 28 hours per week. This suggests that working mothers, despite having a lower amount of time out of work available, may prioritize the time spent with their child over leisure. Therefore, it is important for a model describing the mother's labor supply and child-care decisions to account for the additional choice mothers may make regarding child care and leisure.

FIGURE 1

Maternal child-care time and leisure according to mothers' employment status



NOTES: The vertical axis in the graph on the left represents the fitted values of the following regression:

$$\tau_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it},$$

while the vertical axis in the graph on the right represents the fitted values of the following regression:

$$l_{it} = \beta_0 + \sum_{t=1}^{T} \beta_{1t} t_{it} + \beta_2 d_{it} + \varepsilon_{it}.$$

τ_{it} represents (weekly) maternal time with the child and l_{it} represents leisure time, computed as $l = TT - \tau - h$, where $TT = 112$ is the total time endowment and h represents weekly hours of work. t_{it} represents the child's age fixed effects (with $t = 1, \ldots, 12$), and d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = -7.92$ represents the difference in average maternal time (conditional on the child's age) between working and non-working mothers. $\beta_2 = -28.28$ represents the difference in average leisure time (conditional on the child's age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 572$). For these graphs, the information on a mother's employment status available for the year 1996 has also been used for the year 1997 in order to match it with the mother's child-care time and leisure information. See Section 4 and Appendix B for a description of the dataset.

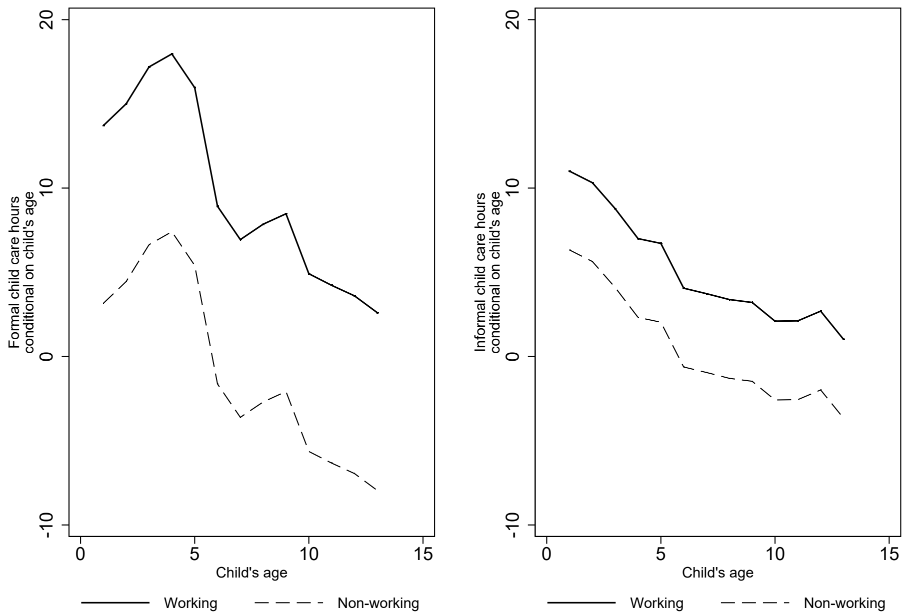
The measure of maternal child-care time used in Figure 1-Left represents the amount of weekly time spent by the child with the mother, so it can be considered a measure of the quantity of maternal time investment. However, mothers may be heterogeneous in the quality of child-care time, which depends on the type of interactions and cognitive stimuli provided to the child. In the psychological literature, Hart and Risley (1995) show that children's IQ scores are strongly associated with the size of their parents' vocabulary. Hence, while it is difficult to directly measure the quality of maternal child-care time in survey data, one may expect the mother's human capital to be a strong determinant not only of her labor market participation decision but also of her productivity in the child's cognitive development process.[8]

As previously mentioned, Figure 1 shows that the difference in maternal time with the child between working and non-working mothers is much smaller than the difference in leisure time. An additional explanation for this pattern can be related to the use of non-parental child care, not only from working mothers but also from the non-working ones. Figure 2 reports the fitted values from two regressions where the dependent variables are formal and informal child care weekly hours regressed on a child's age fixed effects and a binary variable indicating whether the mother works in each period. Formal child care (Figure 2-Left) refers to center-based child care arrangements or family daycare services, while informal child care (Figure 2-Right) refers to the care provided by relatives or nannies. The figure shows that non-parental child care is used by both working and non-working mothers, especially during the child's first years of life. This implies that children of non-working mothers are not available for maternal time investments if cared for in an external setting. However, Figure 2 also confirms that non-parental child care is an important tool to balance work and family when the mother works, as the difference in child-care time between working and non-working mothers is about 11 hours per week for formal child care and about 5 hours per week for informal child care.[9]

[8]Throughout the paper, we distinguish between *high-educated* mothers, with at least some college education (i.e., more than 12 years of education) and *low-educated* mothers, with 12 years of education or less. The mother's child-care quality is not necessarily related to the type of activity performed by the mother with the child. Table B.5 in Appendix B reports the amount of time the mother spends with the child in different categories of activities by distinguishing between high- and low-educated mothers. The table indicates that high-educated mothers and their children are more likely to engage in reading activities than low-educated mothers and their children, but that there are no statistically significant differences in the amount of time spent in other activities based on a mother's level of education.

[9]The negative fitted values for non-working mothers with older children reported in Figure 2 are due to the facts that (i) non-working mothers use non-parental child care for a lower amount of time than

FIGURE 2
Formal and informal child-care time according to mothers' employment status



NOTES: The vertical axis in the graph on the left represents the fitted values of the following regression:

$$f_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it},$$

while the vertical axis in the graph on the right represents the fitted values of the following regression:

$$i_{it} = \beta_0 + \sum_{t=1}^{T} \beta_{1t} t_{it} + \beta_2 d_{it} + \varepsilon_{it},$$

where f_{it} represents (weekly) hours of formal child care, and i_{it} represents (weekly) hours of informal child care in each year t; t_{it} represents the child's age fixed effects (with $t = 1, \ldots, 12$), and d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = 10.55$ represents the difference in average formal child care use (conditional on the child's age) between working and non-working mothers. $\beta_2 = 4.68$ represents the difference in average informal child care use (conditional on the child's age) between working and non-working mothers. The negative fitted values for non-working mothers with older children derive from the facts that (i) they use non-parental child care for a lower amount of time than working mothers, (ii) the use of non-parental child care declines with the age of the child, and (iii) the regressions yield estimated negative constants ($\hat{\eta}_0 = -7.95$ for formal child care and $\hat{\beta}_0 = -3.66$ for informal child care). Source: own elaboration from PSID-CDS data ($N = 2021$). See Section 4 and Appendix B for a description of the dataset.

From Figure 2, two other features emerge that should be taken into account in the model specification. First, the figure suggests a differential use of the two types of child-care services at different ages of the child, with formal child care used for more hours than informal child care before kindergarten age. This pattern may reflect differences in the way mothers perceive non-parental child care services as an investment in children's human capital, with the formal arrangements being more likely to have an educational role than the informal ones. This is also confirmed by previous studies by Bernal and Keane (2011) and Loeb et al. (2007) showing that care in formal arrangements is more likely

working mothers, (ii) the use of non-parental child care declines with the age of the child, and (iii) the estimated regressions yield negative constants.

to boost the child's academic achievement than care in informal settings. Therefore, the model presented below allows different types of child care to have heterogeneous effects on children's cognitive development. Second, the mother may use either a formal or an informal child-care service or a combination of the two for her child at any age. Among the children surveyed in the PSID-CDS data, about 8% receive both formal and informal care before age 6; this proportion decreases to 5% at later ages. For this reason, in the model presented in the next section, mothers are allowed to use at any child's age both formal and informal child care.

3. The model

This section describes the model that is estimated. Section 3.1 presents the basic structure, while Section 3.2 derives the demand functions for all the choice variables. Section 3.3 describes the empirical specification.

3.1. ***Basic structure.*** The model follows a standard framework from Becker and Tomes (1986), in which household preferences are described by a unitary utility function, with a child's ability as an argument, and subject to a production function for a child's ability and budget and time constraints.

The model is dynamic and evolves in discrete time. In each period, the mother decides her own labor supply and time allocation, as well as the amount of non-parental child care to use. In particular, in each period the mother can use both formal and informal child care. The choice variables are then: (i) h_t, representing hours of work; (ii) τ_t, the time the mother spends with the child; (iii) i_t, hours of informal child care; and (iv) f_t, hours of formal child care.[10] The timing is defined as follows: $t = 0$ represents the birth of the child, and the mother makes all the decisions at any child's age t until the child reaches T years of age.[11] The functional form assumptions of the model are based on Del Boca et al. (2014), even though the present model considers a different set of inputs in the child's cognitive ability production function and uses a different empirical specification.[12]

[10]More precisely, in each period the mother decides her labor supply and then chooses the amount of time to spend with the child and the amount of time to use formal and informal child care.

[11]$t = 1$ indicates the first 12 months of the child's life, $t = 2$ refers to the next 12 months of the child's life, and so on. $t = T = 13$ represents the terminal period of the model. It shall be interpreted as the final period of middle childhood before the child enters adolescence.

[12]While the present paper considers the mother's child-care time by distinguishing between high- and low-educated mothers and the amount of time the child spends in formal and informal child care as inputs in the child's cognitive ability production function, Del Boca et al. (2014) consider the mother's and the father's child-care time and the expenditure in goods and services for the child. Concerning

The mother is the unique decision maker in the household concerning the investment choices on the child's human capital.[13] This assumption implies that the father's time allocation is exogenous with respect to the mother's choices and to the child development process.[14] The model applies to intact households where both the mother and the father are present, and only households with one child are considered.[15]

The Mother's Utility Function

The mother's utility in each period is a function of her own leisure time (l_t), that is the time the mother spends alone without working; household consumption (c_t), including the father's and the child's consumption; and the child's cognitive ability (A_t).[16] We assume a Cobb-Douglas form for preferences, and we restrict the preferences parameters to be stable over time:

$$u(l_t, c_t, A_t) = \alpha_1 ln l_t + \alpha_2 ln c_t + \alpha_3 ln A_t \tag{1}$$

where $\sum_{j=1}^{3} \alpha_j = 1$ and $\alpha_j > 0$, $j = 1, 2, 3$.

The mother maximizes her utility subject to the time and budget constraints. The time constraint is defined as:

$$TT = l_t + h_t + \tau_t \tag{2}$$

the empirical specification of the model, this paper improves on Del Boca et al. (2014) by allowing the mother's preferences to be correlated with her unobserved productivity in the labor market. This allows the model to account for the fact that a mother's skills in the labor market may affect her preferences regarding her child's development and vice versa.

[13]To ease the exposition, in the remainder of the paper the mother will be referred to as feminine and the child as masculine.

[14]The model allows the father to affect child development through his labor income, which influences the mother's choices concerning work, formal and informal child care, and time with the child. In the sample of intact households that we use for the estimation of the model, all fathers work, and the average working time does not vary according to the mother's employment status.

[15]In the PSID-CDS data used to estimate the model, the sample of intact households represents 52.7% of the overall sample, and 36.2% of families in the sample have only one child. These figures are in line with US official statistics. According to data from the US Census Bureau referred to the year 2012, 68.1% of children under 18 live with both parents, 66.8% of married women live with their spouses, and 21.6% of married women have only one child. We do not consider single-mother families because they are likely to be characterized by a peculiar decision-making process concerning the choices to work and use external child care, which may also include welfare participation decisions. On the other hand, the inclusion of families with more than one child would require making assumptions about how decisions are made for multiple children and how the productivity of inputs is affected by the presence of siblings. We further discuss the implications of the sample selection in Section 4.

[16]Given that the mother does not care about the child's utility but about the child's cognitive ability in each period, one may interpret these preferences as not properly altruistic but rather paternalistic. The inclusion of a child's ability as an argument instead of the child's utility is a simplifying assumption aimed at avoiding modelling how the child's ability enters in the child's utility function, especially considering the fact that we are dealing with very young children. This is also consistent with other structural papers that adopt a similar specification (see, e.g., Bernal (2008); Del Boca et al. (2014); Mullins (2020)).

where TT is the mother's total time endowment.[17] Note that in each period the mother can choose to spend her leisure time alone (l_t) or to devote some time to the child (τ_t): thus, the model allows the mother to further choose between leisure and time with the child when she is not working.

The budget constraint takes into account household consumption and expenditure for non-parental child care, as well as the total income available in the family (from both parents' labor supply and non-labor income). This is given by:

$$c_t = w_t h_t + I_t - p_{it} i_t - p_{ft} f_t \tag{3}$$

where w_t is the mother's hourly wage; I_t represents the other household earnings (including the father's labor income and the household non-labor income); i_t represents the number of hours that the mother uses informal child care; and f_t represents the amount of time that the mother uses formal child care.[18] Furthermore, p_{it} and p_{ft} represent the hourly price of informal and formal child care, respectively. The model assumes strictly positive prices, implying that services with a potentially zero price in the market (as is the case for most informal arrangements) are characterized by a shadow price representing, for instance, the limited availability of informal care or the value of the unpaid care provider's time in alternative activities (Blau and Currie 2006; Ribar 1992). The mother does not make saving decisions; therefore the other household income defined by I_t is exogenous with respect to all the mother's choices.

It should be noted that no type of child care, despite being measured in terms of weekly hours, has been included in a time constraint or conditioned to the amount of time that the mother works. The current specification in which the amount of external child care is freely determined by the model is preferred because in the data we do not observe a clear pattern in the relationship between a mother's labor supply and non-parental child

[17] $TT = 112$ hours per week. All choice variables are defined on a weekly basis.

[18] In order to keep the model solution and estimation as simple as possible, the specification does not include taxation. Hence, the mother's hourly wage w_t shall be interpreted as the net wage perceived by the mother after taxation is taken into account.

care use that could justify additional assumptions regarding the distribution of these variables.[19]

The Child's Cognitive Ability Production Function

The child's cognitive ability production function (hereafter CAPF) is defined using a value-added specification and takes a Cobb-Douglas form:

$$A_{t+1} = \delta_{0t} \times \tau_t^{\delta_{1t}} \times i_t^{\delta_{2t}} \times f_t^{\delta_{3t}} \times A_t^{\delta_{4t}} \tag{4}$$

where A_{t+1} is the outcome for a child at time $t+1$; τ_t, i_t and f_t are the inputs decided by the mother in each period t, where τ represents the amount of time the mother spends with the child, i the amount of time in informal child care, f_t the amount of time in formal child care, and A_t is the level of the child ability at period t. δ_{0t} represents a total factor productivity (TFP) component, which proxies for the role of missing inputs. As current ability influences the child's future ability, Equation (4) shows that inputs operate with a lag. Moreover, the structure of the CAPF implies that when deciding the inputs in child development, the mother knows the productivity of each of them and the level of the child's ability in the previous period.

The main inputs in the child's CAPF are the amount of time the mother spends with the child and the amount of time the child attends formal and informal child care. The elasticity of a child's cognitive ability with respect to a mother's child-care time δ_{1t} is allowed to vary according to the mother's level of education, to account for the fact that mothers with different levels of education may have heterogeneous effects on the cognitive development of children. The distinction between types of child care allows the production technology to account for the potentially heterogeneous productivity across different child care types, which may also induce differences in a mother' behavior related to their use. The specification of the CAPF allows the mother to use a combination of formal and informal child care in each period, so that the two types of care are not considered mutually exclusive.

[19]In the data used for the model estimation, 49% of mothers use non-parental child care for an amount of time that is slightly lower than their labor supply, while 51% report total non-parental child care use equal to or larger than the mother's labor supply. This implies that mothers may use non-parental child care (i) when they are working because they need someone to look after the child, (ii) for an amount of time that is larger than their labor supply, for example if they think the time spent in non-parental child care can be beneficial to their child's cognitive development, or (iii) for an amount of time lower than their labor supply, for example if other forms of care not directly observed in the data are used for the child.

Despite imposing some limitations on the substitution pattern across inputs because of the assumed functional form,[20] the CAPF specification allows the parameters in (4) to vary across the ages of the child to capture the fact that marginal productivity of inputs varies over the stages of child development (Cunha et al. 2010; Heckman 2007). Equation (4) also includes a TFP component that accounts for inputs not explicitly included in the CAPF, and also varies over time.[21] Finally, a mother's work is not explicitly included in the CAPF because it may not have a direct impact on child development *per se*. A mother's employment may indirectly affect child development through a change in her time allocation, combined with an increase in the use of formal and informal child care.

Maximization Problem

In each period, the mother maximizes her expected lifetime utility, optimally choosing her labor supply, the child care inputs, and the number of hours to spend with the child. In her decision-making process, the mother takes into account the level of ability reached by the child in each period, the wage and child-care price offers that she receives from the market, and the other income available to the household. The child's cognitive ability represents an endogenous state variable, while wage, child care prices, and other household income are exogenous with respect to the maximization problem but differ for each mother in each period. The initial condition of the problem is given by the value of the state variables in the first period.[22]

The value function for the mother at period t is given by:

$$V_t(S_t) = max_{h_t,\tau_t,i_t,f_t} \; u(l_t, c_t, A_t) + \beta E_t V_{t+1}(S_{t+1}) \tag{5}$$

$$s.t. \;\; c_t = w_t h_t + I_t - p_{it} i_t - p_{ft} f_t$$

$$TT = l_t + h_t + \tau_t$$

[20]In fact, it should be noted that, given that the CAPF is Cobb-Douglas, the elasticity of substitution across inputs is always equal to one.

[21]Note that the introduction of the TFP, although making it possible to capture the effects of missing inputs on a child's ability, does not change the mother's optimal investment decisions. Appendix E presents the results of a sensitivity analysis in which we include the father's time with the child among the time investments. Two other important inputs that are omitted in the CAPF specification are (i) the expenditure in goods for the child and (ii) the schooling time from age 5 onward. The main reason for not including them is data availability, as information on both the goods bought by the parents for the child and the amount of time spent by the child at school is available only at one point in time in the period considered for the analysis (i.e., between birth and age 12).

[22]The structure of the initial condition for the child's ability and the draws from which the initial values of w_t, p_{it}, p_{ft}, and I_t are taken will be defined in Section 3.3.

$$lnA_{t+1} = ln\delta_{0t} + \delta_{1t}ln\tau_t + \delta_{2t}lni_t + \delta_{3t}lnf_t + \delta_{4t}lnA_t$$

where the CAPF has been log-linearized for computational convenience, $\beta \in [0, 1]$, and $S_t = \{A_t, w_t, p_{it}, p_{ft}, I_t\}$ represents the vector of state variables.

The maximization problem of the mother can be solved analytically only if the wage offer is exogenous with respect to the mother's past and current labor supply choices. This implies that the wage offer the mother receives in period t is not affected by her working decisions in $(t-1)$ and that it does not reflect any depreciation in the mother's productivity as a result of her absence from the labor market after childbirth. The exogeneity of wages is necessary to estimate the model with continuous choice variables and closed-form solutions, which is needed to allow for four choices and, in particular, to take into account the additional choice between leisure and time with the child.[23]

3.2. ***Terminal period value function and solutions of the model.*** The mother makes her decisions (that are relevant for the child development process described by Equation (4)) in the first T years of the child's life. After period T, both the mother's optimization problem and the child's ability production function change. The mother may continue to optimally choose labor supply and consumption, but she will no longer consider maternal and non-parental child care choices. The terminal level of a child's cognitive ability is A_{T+1}, that is the level of ability reached in $T+1$ that will not be affected by the mother's subsequent decisions. This level of ability may be interpreted as the starting point for the child's future development during adolescence, from $T+1$ on.

The period $T+1$ maximization problem for an infinitely lived household can be written as:

$$V_{T+1} = \tilde{V}_{T+1} + \sum_{\kappa=0}^{+\infty} \beta^{\kappa} \alpha_3 lnA_{T+1} \tag{6}$$

where

$$\tilde{V}_{T+1} = max_{h_{T+1}}\ \alpha_1 lnl_{T+1} + \alpha_2 lnc_{T+1} + \beta E_{T+1}\tilde{V}_{T+2}(l_{T+2}, c_{T+2})$$

[23]However, this assumption may have implications for the estimation results and the fit of the model. In fact, because the wage process does not take into account the potential decrease in wages when leaving the labor market after childbirth, mothers may find it profitable to stay out of the labor market more than they would do in the case of endogenous wages. Therefore, the model may overestimate the proportion of mothers not working and underestimate their labor supply, especially during the child's early years of life (see also Section 6.1).

and $\sum_{\kappa=0}^{+\infty} \beta^{\kappa} = \rho$ represents the value given by the mother to the child's ability in the last period.[24] Equation (6) represents the terminal period value function and implies that the mother's maximization problem after period T becomes stationary and does not depend on the choices she made in the previous periods.

The model is solved by backward induction and yields closed-form solutions for all the choice variables. The solution of the model involves the computation of the value function starting from the terminal period and the corresponding optimal solutions in each period. Following a two-stage process, we first derive the optimal solutions for maternal time (τ_t) and non-parental child care (i_t and f_t), conditional on h_t, and then compute the solutions for the mother's labor supply h_t. The analytical derivation of the results is reported in Appendix A.

The demands for maternal child-care time and non-parental child care conditional on the mother's labor supply for any period t are given by:

$$\tau_t^c = \frac{\beta\delta_{1t}D_{t+1}}{(\alpha_1 + \beta\delta_{1t}D_{t+1})}(TT - h_t) \tag{7}$$

$$i_t^c = \frac{\beta\delta_{2t}D_{t+1}}{p_{it}(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{8}$$

$$f_t^c = \frac{\beta\delta_{3t}D_{t+1}}{p_{ft}(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}(w_t h_t + I_t) \tag{9}$$

where $D_{t+1} = \frac{\partial V_{t+1}}{\partial lnA_{t+1}}$ represents the marginal utility the mother gets from the child's future cognitive ability in each period. The sequence of marginal utilities from period $T+1$ to period 1 is given by:

[24]In the estimation, the discount factor is set at $\beta = 0.95$. To increase the flexibility of the model and to allow the discount factor of the mother to differ in the last period of investments with respect to the previous ones, the parameter ρ is estimated.

$$
\begin{aligned}
D_{T+1} &= \rho\alpha_3 \\
D_T &= \alpha_3 + \beta\delta_{4T} D_{T+1} \\
D_{T-1} &= \alpha_3 + \beta\delta_{4T-1} D_T \\
&\vdots \\
D_t &= \alpha_3 + \beta\delta_{4t} D_{t+1} \\
&\vdots \\
D_2 &= \alpha_3 + \beta\delta_{42} D_3 \\
D_1 &= \alpha_3 + \beta\delta_{41} D_2
\end{aligned}
\tag{10}
$$

Appendix A reports the first-order conditions used to derive the demands for all inputs. In particular, the first-order condition for maternal child-care time (Equation (A.2) in Appendix A) shows that the marginal cost of maternal time corresponds to the marginal utility the mother derives from leisure. Specifically, by defining $\bar{V}_t = \alpha_1 ln(TT - h_t - \tau_t) + \alpha_2 ln(w_t h_t + I_t - p_{it} i_t - p_{ft} f_t) + \alpha_3 ln(A_t)$ as the current utility in period t, the mother's marginal utility from leisure is given by $\bar{V}_t'(l) = \frac{\alpha_1}{TT - h_t - \tau_t}$. This expression indicates that the cost of maternal time investment is greater for working mothers (with $h_t > 0$) who face a higher opportunity cost of spending time with their children. If a mother does not work (hence $h_t = 0$), the cost of maternal child-care time becomes $\bar{V}_t'(l) = \frac{\alpha_1}{TT - \tau_t}$, and it only depends on the mother's preferences for leisure α_1.

Equations (8) and (9) indicate that the demand for non-parental child care can be driven by the necessity of custodial care, that is, if the mother is working and needs someone to look after the child, or by valuing the educational role of the service. In fact, non-working mothers (for which $h_t = 0$) may demand non-parental child care if they value their children's ability and they think child care may represent an input for their children's development, as long as the other household income is strictly positive and sufficiently high.

An implication of the Cobb-Douglas specification used in the child's CAPF is that all inputs should be strictly positive, but we do allow for the possibility of corner solutions for the mother's labor supply decisions.[25] The mother's latent labor supply, conditional on τ_t^c, i_t^c and f_t^c, is given by:

$$h_t^c = \frac{\alpha_2(TT - \tau_t^c)}{\alpha_1 + \alpha_2} - \frac{\alpha_1(I_t - p_{it}i_t^c - p_{ft}f_t^c)}{w_t(\alpha_1 + \alpha_2)} \tag{11}$$

Substituting (7) , (8), and (9) into Equation (11), the latent labor supply becomes:

$$h_t^* = \frac{TT(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})}{(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})} - \frac{I_t(\alpha_1 + \beta\delta_{1t}D_{t+1})}{w_t(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})} \tag{12}$$

The actual labor supply in each period is determined according to the following rule:

$$\mathrm{h}_t = \begin{cases} h_t^* & \text{if} \quad h_t^* > 0 \\ 0 & \text{if} \quad h_t^* \leq 0 \end{cases}$$

The mother's latent labor supply is negative or zero if the wage offer the mother receives in any period is below her reservation wage, which is given by:

$$w_t^* = \frac{I_t(\alpha_1 + \beta\delta_{1t}D_{t+1})}{TT(\alpha_2 + \beta\delta_{2t}D_{t+1} + \beta\delta_{3t}D_{t+1})} \tag{13}$$

Equation (13) shows that a mother's reservation wage increases with the other household income I_t, a mother's preferences for leisure α_1, and the elasticity of a child's ability with respect to a mother's child-care time δ_{1t}. The reservation wage is instead negatively affected by the elasticity of a child's ability with respect to the alternative forms of care (δ_{2t} for informal child care and δ_{3t} for formal child care.

3.3. ***Empirical specification of the model.*** Unobserved and observed heterogeneity enters any stage of a mother's decision-making process. Consider first the mother's utility function, where the parameters, because of the functional form assumptions, should be positive and sum to one. To respect these requirements without imposing additional constraints on the estimation algorithm, we use a suitable transformation of the original parameters. More precisely, we allow the coefficients in the mother's utility function to vary according to unobserved taste shifters representing the utility from consumption (γ_2) and the utility from the child's ability (γ_3). Therefore, the parameters representing

[25]Ideally, one may want to allow for corner solutions in both labor supply and non-parental child care decisions. In the data used to estimate the model, the proportion of children using formal care is about 50%, and the proportion using informal child care is 22%. The functional form of the production technology has been chosen mainly for model tractability, as a model allowing for corner solutions in both labor supply and non-parental child care would be intractable.

the mother's preference for leisure (α_1), consumption (α_2), and the child's ability (α_3) are defined as:

$$\alpha_1 = \frac{1}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{14}$$

$$\alpha_2 = \frac{exp(\gamma_{2k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{15}$$

$$\alpha_3 = \frac{exp(\gamma_{3k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{16}$$

where γ_2 and γ_3 follow a discrete distribution with two points of support ($k = h, l$).

In each period, the mother receives a wage offer and decides whether to enter the labor market by comparing the value of this offer with the reservation wage defined by Equation (13). The offer the mother receives is described by the following wage equation:

$$ln(w_t) = \mu_t + \epsilon_t \tag{17}$$

where

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma_\epsilon^2)$$

is assumed to be uncorrelated over time and represents a transitory shock to the wage. The term μ_t is the mean of the log wage draws of the mother at time t and is defined as follows:

$$\mu_t = \mu_{mk} + \mu_1 Edu + \mu_2 Age_t + \mu_3 Race + \mu_4 Cohort + \mu_5 MacroArea + \mu_6 Cohort \times MacroArea \tag{18}$$

where Edu represents a mother's years of education; $Race$ is a dummy variable equal to one if the mother is white; $Cohort$ indicates the mother's year of birth; and $MacroArea$ reports the geographical area where the mother lives.[26] The interaction term between $Cohort$ and $MacroArea$ captures differences in the wage opportunities for mothers who belong to the same cohort but live in different geographical areas.

[26]The variable $MacroArea$ is a binary indicator equal to 1 if the mother lives in the Northeast region of the US, where wages are observed to be higher than in the rest of the country, and 0 otherwise.

The component μ_{mk}, where $k = h, l$, represents the mother's unobserved skills in the labor market that are allowed to be correlated with the mother's preferences. The specification of the model assumes that the mother's unobserved productivity and her preferences regarding the child's ability follow a bivariate discrete distribution (Heckman and Singer 1984) with two points of support. This determines four types of mothers, identified by their level of productivity in the labor market and by their level of preference for the child's ability. The probability that a mother belongs to each type should be estimated.[27]

Similarly to the wage process, the income process is exogenous with respect to the mother's input decisions in each period. The other household income is assumed to evolve according to a lognormal distribution and to depend on the father's observable characteristics and a shock:

$$ln(I_t) = \mu_{inc0} + \mu_{inc1}FatherEdu + \mu_{inc2}FatherRace + \mu_{inc3}FatherAge_t + \iota_t \tag{19}$$

where $\iota_t \overset{\text{iid}}{\sim} N(0, \sigma^2_{inc})$.[28]

As reported in Equations (8) and (9), the demands for informal and formal child care depend on the price of both types of service. The data used for the estimation of the model provides information on the hourly price paid by the mother for each arrangement (either formal or informal) but presents an empirical challenge, which is given by the fact that a sizable proportion of mothers report a zero price.[29] This could be because the child is cared for in an informal setting and the service is provided for free (as is

[27]It should be noted that the unobserved skills type of the mother μ_{mk} is fixed over time and therefore represents the only form of persistence in the wage process allowed in the model. While adding serial auto-correlation in the wage process would still make the model tractable, the identification of the auto-correlation coefficient would require using exactly the same variation in the data that is already used to identify the mother's unobserved type, that is the wage correlation over time.

[28]Given that the other household income evolves separately from the mother's wage, the model does not account for potential positive assortative matching in the marriage market. Positive assortative matching may have two main implications. First, a mother's labor market opportunities may be correlated with her husband's, and therefore her labor supply decisions may also depend on the husband's labor supply. However, the data do not show significant variation in the father's labor supply across the mother's employment status and by a child's age. Second, mothers and fathers may positively match not only according to their labor market opportunities but also in terms of their productivity in the child's development process so that the current specification of the model may overestimate the role played by the mother's child-care time. In relation to this, Appendix E presents the results of a sensitivity analysis in which the time investment received by the child includes both the mother's and the father's child-care time and shows that the estimated elasticity of a child's cognitive ability with respect to a mother's child-care time is not affected.

[29]In the data used to estimate the model, this occurs for 38.17% of mothers for formal child care and 49.70% of mothers for informal child care. See also Section 4.

usually the case with informal care provided by grandparents or other family members), or because the mother reports the out-of-pocket price of formal child care services without mentioning potential vouchers or subsidies. Given that mothers reporting a zero price do not represent a random portion of the sample, we aim to solve this selection issue by specifying two child care cost equations in which the price of formal and informal child care is described by variables that are exogenous to the mother's decision-making process. For formal child care, such a variable is represented by the amount of funding that each US state allocates to pre-kindergarten, which is taken from the National Institute for Early Education Research (NIEER 2003) and refers to 3- and 4-year-old children. For informal child care, we use an additional variable from the CDS data which asks the primary caregiver whether other family members live in the same neighborhood and therefore might be available to take care of the child in an informal setting. The child care cost equations are specified as follows:

$$p_{it} = exp(\lambda_{i0} + \lambda_{i1} I[family] + \epsilon_{it}) \tag{20}$$

$$p_{ft} = exp(\lambda_{f0} + \lambda_{f1} StateFunding + \epsilon_{ft}) \tag{21}$$

where p_{it} and p_{ft} represent the cost of informal and formal child care, and the exponential forms ensure that such costs are positive.[30]

Concerning the child's CAPF, as stated in Section 3.1, the parameters vary according to a child's age, and the parameter representing the elasticity of a child's ability with respect to a mother's child-care time is allowed to vary according to the mother's level of education. Specifically, they are defined as follows:

[30] Other structural papers have adopted two alternative strategies to tackle the non-random reporting of zero child-care price. The first strategy consists of estimating the (time-invariant) prices of formal and informal child care as if they were parameters of the model without using the data (see, e.g., Bernal (2008)). This would be problematic in our setting because the identification of these parameters should be obtained by exploiting the time-invariant correlation between choices and monetary variables (i.e., a mother's wage and other household income), which is also used to recover the preference parameters that are fixed over time. The second approach uses only the observations reporting a positive price (see, e.g., Caucutt et al. (2020)), but in our case this would imply keeping only about 60% of the sample for the estimation of the parameters related to the formal child-care cost, and about 50% of the sample for the parameters related to the informal child-care cost. For these reasons, we prefer to keep the entire sample and to solve the issue of zero prices by specifying the child-care cost equations.

$$\delta_{0t} = exp(\xi_{0tfp} + \xi_{1tfp} \times t) \tag{22}$$

$$\delta_{1t} = exp(\xi_{0\tau} + \xi_{1Edu} \times HighEduMom + \xi_{1\tau} \times t) \tag{23}$$

$$\delta_{2t} = exp(\xi_{0i} + \xi_{2i} \times t) \tag{24}$$

$$\delta_{3t} = exp(\xi_{0f} + \xi_{3f} \times t) \tag{25}$$

$$\delta_{4t} = exp(\xi_{0A} + \xi_{4A} \times t) \tag{26}$$

where t indicates the age of the child, and $HighEduMom$ is a binary variable equal to 1 if the mother has more than 12 years of education.

To estimate the model and to take into account the dynamic optimization problem faced by the mother, it is necessary to know the starting level of ability, that is the child's cognitive ability the mother observes in the first period of investments. The initial ability endowment is assumed to be a function of the observed characteristics of the child and the mother at childbirth. Specifically:

$$A_{1i} = exp(\eta_0 + \eta_1 BirthWeight_i + \eta_2 Male_i + \eta_3 MotherAgeBirth_i + v_i) \tag{27}$$

where $BirthWeight$ is a dummy variable indicating if a child has a birth weight lower than 2500 grams, $Male$ is a dummy variable indicating whether the child is a male, and $MotherAgeBirth$ indicates the age of the mother at the birth of her child. The choice of the characteristics to be included in Equation (27) is driven by the existing evidence that these characteristics are correlated with the child's initial skills endowment. In particular, medical research has shown that being born with a birth weight lower than 2500 grams increases the risk of developmental problems (Hack et al. 1995) and that children born to a teenage mother are likely to be less healthy (Lopez 2003). The parameter η_0 is a constant, while $v \overset{\text{iid}}{\sim} N(0, \sigma_v^2)$ is a shock representing the variation in initial ability not captured by the observed characteristics.[31]

Recalling the value-added specification of Equation (4), the estimation provides consistent estimates of the parameters in the CAPF for each input if the following conditions

[31] Other structural papers, such as that by Bernal (2008), have adopted a similar strategy. Del Boca et al. (2014) retrieve the initial ability of the child from the first test scores observed in the data, but we decide not to follow this approach because it would require keeping only children with at least two test scores, and the sample would reduce to 181 observations.

hold: (i) A_t is a sufficient statistic for all the inputs received by the child in the previous periods; (ii) the child's initial endowment A_1 (that the mother observes but the researcher does not) only affects the mother's decisions in the subsequent period and does not affect child's ability in the future periods (Todd and Wolpin 2003). Considering the specification of the child's initial ability outlined in Equation (27), the latter condition implies that the characteristics used to proxy the observable component of the initial skills endowment only impact the child's level of ability in the first period and do not have an effect on subsequent periods' investments and ability.

Finally, it should be described how the child's true cognitive ability is related to the measure of that given by the test scores. The score measures used in the empirical analysis are the Letter Word (LW) and the Applied Problems (AP) raw scores, which are simple sums of the number of questions answered correctly by the test taker. Following the approach based on classical test theory (Novick 1966), and also adopted by Del Boca et al. (2014), we define the probability that the child answers each item correctly as follows:

$$\pi_{score} = \frac{exp(ln(A_t + \kappa LW))}{1 + exp(ln(A_t + \kappa LW))} = \frac{A_t + \kappa LW}{1 + A_t + \kappa LW} \tag{28}$$

where A_t is the child's true cognitive ability, and LW is a dummy variable indicating whether the test score is the LW raw score, which captures the differences in the item difficulty between the LW and the AP scores. The final test score is distributed as a Binomial random variable, with parameters (J_t, π_{score}), where J_t is the maximum number of items in the test.[32] This specification properly accounts for measurement error in the test score measures, as a child's scores may not perfectly reflect his true cognitive ability.

4. Data

This paper uses data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS) and Time Diary (TD) components. The PSID is a longitudinal study that began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the US. Starting from 1968, information about each family member was collected, but much greater detail was obtained about the

[32]In the empirical application, $J = 57$ for both the LW and the AP scores. The specification allows the LW and the AP scores to differ based on the presence of the κLW component in the probability of answering each item correctly, as well as from a different stochastic process from which the two test scores errors are drawn.

head and the spouse. From 1997, the CDS has gathered information on children aged 0-12 in PSID families through extensive interviews with their primary caregivers. The CDS was replicated in 2002 and 2007 for children under 18.

For this analysis, we exploit the child cognitive ability measures and non-parental child-care data provided in the Primary Caregiver Interview of the CDS, together with the time use details given in the TD component of the CDS.

The CDS asks the primary caregiver about the non-parental child-care arrangements used for the child from birth until kindergarten and at the time of the survey in the case the child is beyond kindergarten age. More precisely, the mother can report more than one arrangement used in each period and is asked to indicate the type of arrangement, as well as the weekly amount of time it is used and its hourly price. For the analysis, we define the formal and informal child-care variables by exploiting the information on the formal and informal arrangement used more frequently for every age of the child. The formal category includes family daycare and preschool, while the informal category includes care provided by relatives, non-relatives and babysitters. The same distinction applies when the child reaches school age. Formal arrangements include any type of before- or after-school programs or any other kinds of center-based setting that the child may attend outside of school time (e.g., extra-curricular activities, sport, training sessions), while the informal arrangements include relatives or nannies. Using the 1997, 2002, and 2007 waves, we can recover the complete child-care history (from birth until kindergarten) of the sampled children, as well as information on the formal and informal arrangements that they use at the time of the survey.

The CDS supplement also provides several measures of child cognitive skills based on the Woodcock Johnson Achievement Test Revised (WJ-R) (Woodcock and Johnson 1989). The outcome measures considered in this study are the LW and the AP test scores that are acquired for all children older than four and that prove, respectively, a child's learning and reading skills and a child's skill in analyzing practical problems in mathematics (Hoffert et al. 1997). These measures are available for the survey years 1997, 2002, and 2007.

In 1997 and 2002, the CDS includes another instrument to assess the time use of children, the TD, which consists of a chronological report about the child's activities over a specified 24-hour period.[33] Each participating child completes two time diaries, one for a weekday (Monday-Friday) and one for a weekend day (Sunday or Saturday). The TD additionally collects information on the social context of the activity by specifying with whom the child was doing the activity. The variable *weekly time with the mother* is constructed by multiplying the daily hours the child spends with the mother by five for the weekday and by two for the weekend day and summing up the total hours in a week.[34]

The main PSID surveys are used to gather information about the labor supply of mothers and fathers and the household non-labor income. PSID interviews were conducted annually until 1997, and since then they have been biennial. As children in 1997 have different ages, ranging from 0 to 13, and in order to identify the necessary information for all of them at every age, CDS data should be matched with family information from PSID surveys in the years 1985-2007.[35] The family information we gather includes each parent's hours of work, wage, and non-labor income in each period.[36]

All relevant variables are constructed for every age of every child, defining age one as the first 12 months of a child's life, age two as the next 12 months, and so on.[37] For

[33]The primary caregiver completes the time diary for the very young children, while older children are expected to complete the time diaries themselves (ISR 2010a,b).

[34]As anticipated in Section 2, we consider the total time spent by the child with the mother without conditioning on any specific activity. This is also consistent with the model specification outlined in Section 3, according to which the mother allocates her time between work, leisure (alone), and time with the child. Only the latter is productive for the child's human capital accumulation process.

[35]For instance, to identify household information for all relevant periods for a child born in 1996 (1 year old in 1997) we need to use PSID surveys in the period 1997-2007. If a child is born in 1986 (aged 11 in 1997) we need to use PSID surveys in the period 1987-1999. All PSID surveys in the period 1985-2007 were exploited, and the children included in the final sample were born between 1984 and 1996. See Appendix B, Tables B.1 and B.2.

[36]Note that all the variables that we use from the main PSID surveys concerning labor and non-labor income of the household members refer to the year before the survey. All monetary variables are deflated into 1997 US$ using the Consumer Price Index (CPI) History for the US. The variables used to recover the mother's wage in every period refer to after-tax labor income. See Appendix B for a further description of the data sources used for the analysis.

[37]There may be a discrepancy between a child's age that depends on the child's date of birth and the calendar year to which the variables from the main PSID survey refer, such as the mother's wage. This mismatch is likely to matter more for children born toward the end of the calendar year (e.g., from August until December) and to be particularly relevant for the mother's wage information. In fact, it could be the case that mothers of children born toward the end of the calendar year are assigned, in a systematic way, the wage they were earning the period before. In the first period, the assigned wage may misleadingly refer to the wage the mother was earning before the birth of the child, and therefore may overestimate the wage earned in the first 12 months of the child's life. However in the data we observe that the wages of mothers giving birth between August and December are not systematically lower or higher than the wages of mothers who gave birth earlier, and that the first period wage of mothers who gave birth toward the end of the calendar year is slightly smaller (and not greater) than the wage of mothers who gave birth at the beginning of the year, thus excluding that the mismatch overestimates the mother's wage when the child is aged one.

the estimation of the model, we consider children interviewed in the first wave of the CDS who live in intact households (where both the mother and the father are present), who do not have any siblings, and without missing data concerning personal and parents' demographic characteristics. The final sample is made up of 417 observations.

Table 1 shows the average values of all the variables for the sample. On average mothers work 27 hours per week and spend 21 hours per week with their child. Formal child care is used on average 10 hours per week, and this value is larger than the amount of time informal child care is used. The mother's hourly wage is, on average, 14 US$, while other household income amounts to, on average, around 800 US$ per week. The average price of formal and informal child care is, respectively, 1.08 and 0.28 US$.[38] Finally, the average LW score in the sample is around 35 out of 57, while the AP score is around 30 out of 52.

Table 1
Descriptive statistics of all variables for the entire period

	Mean	SD	Min	Max
Child's LW raw score	35.10	14.46	1	57
Child's AP raw score	29.62	10.53	1	52
Mother's hours of work	27.30	17.53	0	100
Non-working mother	0.19	0.39	0	1
Mother's time with the child	21.16	17.01	0.17	95.75
Formal child care	10.26	16.92	0	70
Informal child care	5.84	13.26	0	60
Mother's wage	14.36	10.27	5.01	133.92
Other household income	791.36	644.15	0.09	8834.95
Price of formal child care*	1.08	3.60	0	72
Price of informal child care*	0.28	1.29	0	33.33
Child's gender: male	0.51	0.50	0	1
Child's birth weight	3387.15	614.56	907.18	6917.28
Mother's age at child's birth	28.20	5.10	16	43
Mother's education	13.27	2.48	2	17
Mother's race: white	0.61	0.49	0	1

NOTES: Monetary variables deflated into 1997 US$. Mother's hours of work, formal and informal child care hours, mother's time with the child, and other household income are weekly values. Mother's wage and the price of formal and informal child care are hourly values. The child's birth weight is expressed in grams. Other household income includes the father's labor income and household non-labor income.
* The price of formal and informal child care includes zero values, which are reported by 38.17% and 49.70% of the sample, respectively. The average prices of formal and informal child care without the zeros are, respectively, 3.46 and 2.52 US$ per hour, while the minimum prices of formal and informal child care without the zeros are, respectively, 0.5 and 0.25 US$ per hour.
Source: own elaboration from PSID-CDS data.

In what follows, we discuss what biases might be introduced into the analysis by focusing on the subsample of children living in intact households without siblings. This sample selection implies that all mothers' investments in their children's ability are unrelated to

[38] It should be noted that these variables also include zero values. The average prices of formal and informal child care without the zeros are, respectively, 3.46 and 2.52 US$ per hour, while the minimum prices of formal and informal child care without the zeros are, respectively, 0.5 and 0.25 US$ per hour.

the decision to marry or to cohabit and to fertility. However, if mothers in intact households have a more marriage-oriented attitude, which also influences their time allocation and fertility, they may be more likely to stay at home and to spend more time with their children instead of working. This may lead to an overestimation of the proportion of mothers not working or to an overestimation of the mothers' preferences regarding their children's ability. Similarly, mothers with only one child may have higher preferences regarding their children's ability and this may lead to an overestimation of the mother's use of the most productive input. However, women in long-term relationships and with fewer children may also be more desirable in the labor market. In addition, the fact of having only one child means that the mother has experienced only one work interruption as a result of childbirth, thus making the sample disproportionately represented by highly productive mothers and leading to an overestimation of a mother's attachment to the labor market. These arguments suggest that the sample selection may oversample mothers who are more productive either in the labor market or at home with their children.[39]

5. Estimation and Identification

The model parameters are estimated using a Method of Simulated Moments estimator that minimizes the distance between several data statistics and their model counterparts. The full list of statistics used for the estimation appears in Table C.1 in Appendix C.

By following the data-generating process implied by the model described in Section 3, we simulate the same statistics for the individuals (mothers and children) in the sample over the child's life cycle. The simulation is obtained by taking $N \times R$ random draws from the initial distribution implied by the model, that is, the shock in the child's initial ability, the mother's skills and type preference distributions, and, for each period, the wage, the child-care prices, and the other income distributions.[40] After having drawn the child's level of ability, the wage offer, the child-care prices, and the level of other income in the first period, the optimal choices of the mother are obtained by exploiting

[39]Table B.4 in Appendix B compares the characteristics of the subsample used for the analysis ($N = 417$) with the ones of the entire PSID-CDS sample ($N = 3243$). It shows that mothers in this subsample are, on average, older and more educated, work more, use more non-parental child care (both formal and informal), and spend less time with their children, than mothers in the entire sample. However, the wage before childbirth of the mothers in the subsample is not statistically different from that of mothers in the entire sample.

[40]$N = 417$ and $R = 5$.

the optimal solutions derived in Section 3.2.[41] This process is repeated for every period up to the final one T. The simulated data are used to compute the same statistics defined in Table C.1. Both actual and simulated statistics are used to construct the objective function to be minimized. The Method of Simulated Moments estimator is then:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)' W \hat{g}(\theta) \tag{29}$$

where $\hat{g}(\theta) = \hat{m} - \hat{M}(\theta)$, $\hat{m}$ is the vector of statistics defined from the actual data, and $\hat{M}(\theta)$ is the vector of simulated statistics according to the model.[42] Given S number of moments, the weighting matrix is defined as:

$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap. The standard errors are also computed with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. Appendix C provides further details on the estimation.

The estimation requires a unique solution for the minimization of the objective function, which in practice depends on the uniqueness of the minimum and on the curvature around it. Formally testing for this is not feasible, as it would require accounting for the multidimensional nature of the parameters space. However, we can represent the pattern of the objective function whenever we vary each parameter one by one. Figure C.1 in Appendix C shows that the value of the objective function varies when perturbing each parameter from its estimated value, thus suggesting that this seems to be the case.

The identification of the model parameters relies on parametric and functional form assumptions, on exclusion restrictions (that is, variables entering in some parts of the model and not in others), and on the choice of the moment conditions. The model assumptions and exclusion restrictions are presented in Section 3. The choice of the

[41]To numerically test the accuracy of the solutions given by the theoretical model, we also perform a grid search, assuming that the mother's decision to work was actually discrete. In other words, we compute the value of the demands for formal and informal child care and time with the child, and the mother's inter-temporal utility for different levels of the mother's labor supply (with the number of hours of work ranging from 0 up to the total time endowment) and we define as optimal choices those that provide the highest utility. The solutions do not differ from the ones provided by the theoretical model, although the computation becomes more time consuming.

[42]The estimation is done using the simplex algorithm, which is robust to non-smooth objective functions, by setting a smaller step function than the routine's default.

moment conditions requires that the statistics listed in Table C.1 are informative of the corresponding parameters, in such a way that a slight perturbation of the parameters results in a variation of the moments value. In what follows, we describe the specification of the most relevant moment conditions and show that they vary whenever we perturb the corresponding parameters of the model.

To identify the mother's labor market opportunities, which are proxied by the wage equation, we exploit the variation in wages over the mother's life cycle, across cohorts, and between geographical areas, by accounting for the macro-area where a mother lives. The wage offer is also a function of the mother's productivity in the labor market and of a transitory shock. To identify these unobservable components, we use the residuals from an ordinary least squares (OLS) regression of the mother's wage on education, race, cohort, area of residence, and the interaction between the latter two. The variance of these residuals captures the variation of both the time-invariant mother's unobserved productivity and the time-varying shock. By regressing the residuals in each period onto their lagged value we get a moment that depends only on the persistence of the mother's unobserved productivity. Considering that mothers are grouped in four categories defined by their level of productivity in the labor market and by their level of preference for the child's ability, Figure C.2 in Appendix C reports how the perturbation of the estimated proportion of mothers in each category affects the variance and the autocorrelation of the wage residuals.

For the estimation of the parameters in the formal and informal child-care cost equations, we use the correlation between the price of each type of service and its own determinant as a moment. That is, we correlate the price of formal child care with state funding for center-based child care and the price of informal child care with the presence of family members in the neighborhood. Figure C.3 in Appendix C shows that both moments vary whenever we perturb from their estimated values the parameters λ_{i1} and λ_{f1} in Equations (20) and (21).

The parameters in the child's CAPF are identified by the correlation between a mother's choices in t and the child's test scores in $t+1$.[43] Concerning the contribution of the

[43] Due to the structure of the data, when defining the moment for the elasticity of a child's cognitive ability with respect to a mother's child-care time, we use as an outcome the test scores observed in the next survey, that is, after five years. For the specification of all moments, the test scores refer to both the LW and the AP scores.

mother's level of education to the elasticity of her child's ability with respect to her own child-care time (ξ_{1Edu} in Equation (23)), it should be taken into account that a mother's education also enters in the wage equation. Therefore, we use as a moment the coefficient of a mother's level of education in an OLS regression on test scores, where we also control for the mother's wage, to partial out the effect of education on the mother's labor market productivity. Figure C.4 in Appendix C reports the variation in the moment conditions used to identify $\xi_{1\tau}$ and ξ_{1Edu} in Equation (23) that we obtain when we vary the estimated parameters. Figure C.5 reports the same variation obtained with the moments on formal and informal child care used to identify the parameters ξ_{2i} in Equation (24) and ξ_{3f} in Equation (25).

The fact that a mother's education enters into the CAPF also affects the way mothers with different educational levels make investment decisions. We identify this differential behavior by regressing maternal child-care time and labor supply on the mother's level of education, and by using the coefficient of a mother's education as a moment. Figure C.6 in Appendix C shows that these moment conditions are affected by variations in the parameter ξ_{1Edu} and can thus be used to identify the way mothers with a different education level substitute their time with the alternative care provider's time.

Finally, the estimation of the model crucially relies on the identification of the initial condition for the child's level of ability. As reported in Equation (27), the child's initial ability is specified as a function of some observable characteristics of the child and of the mother at childbirth. To estimate the contribution of each characteristic, we can use as moments their correlation with the child's test scores. However, we shall also take into account that the data do not provide a measure of cognitive ability before age four, and that for some children the observed scores may refer to later ages. Therefore, we define these moments by using only the first test scores observed for each child and by taking the residuals from an OLS regression of such scores on a child's age fixed effects to partial out any age effects. Table C.2 in Appendix C reports the results of two regressions on the characteristics listed in Equation (27) in which we use the raw test scores (Column 1) and the residuals (Column 2) as dependent variables. The coefficients in Column (2) are either more statistically significant or have lower standard errors than the coefficients reported in Column (1). This suggests that building the moments with the residuals rather than with the raw test scores improves the identification of the parameters in the child's initial level of ability. Finally, the variance of these residuals is used to recover the error variation in the initial ability; the variation in the corresponding moment condition is reported in Figure C.7 in Appendix C.

6. Results

This section presents the main results of the model estimation by discussing the parameters in the mother's utility function, the wage and child-care cost equations, and the estimated parameters for maternal time and non-parental child care in the CAPF.[44]

Panel A of Table 2 reports the preference parameters for leisure (α_1), consumption (α_2), and a child's ability (α_3) for each one of the four subgroups in the sample, which are defined by the levels of preference for consumption (γ_2) and a child's ability (γ_3), according to Equations (14), (15), and (16). Type I corresponds to a low level, while Type II corresponds to a high level. The results show that for all preference parameters the largest variation across the four groups is induced by the utility from a child's ability: Type II mothers (with γ_{3h}) have values of α_1 and α_2 that are 13% lower and values of α_3 that are 53% larger than Type I mothers (with γ_{3l}). This implies that mothers with a low level of utility from their child's ability (that is, belonging to Type I, or with γ_{3l}) have higher preferences for both leisure and consumption.

The model allows the preference parameter for a child's ability to be correlated with the unobserved skills of the mother in the labor market (μ_m), which are similarly discrete. The estimated skills levels, reported in Panel B of Table 2, show that there are not large differences between high- and low-skilled mothers, although only the skill level for the High type μ_h is statistically different from zero. According to Table 3, almost 44% of mothers in the sample belong to the Low type.

Table 2 also reports the estimated value for the parameter ρ (bottom of Panel A), indicating the weight that the mother puts on the child's level of ability reached in the last developmental period, which is estimated to be 44.[45] Panel B of Table 2 lists the estimated parameters in the wage equation; almost all of them have the expected sign and reasonable magnitude, although not all of them are statistically significant.

[44]The remaining estimated parameters are reported in Appendix D.

[45]As Del Boca et al. (2014) point out, having found a discount factor in the last period larger than the one we could get by fixing it to the value assigned to β (i.e., $\beta = 0.95$ so that $\rho = \sum_{\kappa=0}^{+\infty} \beta^{\kappa} = \frac{1}{(1-\beta)} = 20$) implies that the mother gives a lot of importance to the level of ability that the child reaches in the final period.

Table 2
Estimated parameters in the mother's utility function and the wage equation

		Estimate	Std. Errors
Panel A. Utility function			
$\alpha_{1\gamma_{2l}\gamma_{3l}}$	Preference for leisure (Type I consumption, Type I child ability)	0.4037	0.0230
$\alpha_{1\gamma_{2l}\gamma_{3h}}$	Preference for leisure (Type I consumption, Type II child ability)	0.3494	0.0192
$\alpha_{1\gamma_{2h}\gamma_{3l}}$	Preference for leisure (Type II consumption, Type I child ability)	0.4030	0.0594
$\alpha_{1\gamma_{2h}\gamma_{3h}}$	Preference for leisure (Type II consumption, Type II child ability)	0.3490	0.0243
$\alpha_{2\gamma_{2l}\gamma_{3l}}$	Preference for consumption (Type I consumption, Type I child ability)	0.3949	0.0510
$\alpha_{2\gamma_{2l}\gamma_{3h}}$	Preference for consumption (Type I consumption, Type II child ability)	0.3419	0.0443
$\alpha_{2\gamma_{2h}\gamma_{3l}}$	Preference for consumption (Type II consumption, Type I child ability)	0.3959	0.0518
$\alpha_{2\gamma_{2h}\gamma_{3h}}$	Preference for consumption (Type II consumption, Type II child ability)	0.3427	0.0600
$\alpha_{3\gamma_{2l}\gamma_{3l}}$	Preference for child ability (Type I consumption, Type I child ability)	0.2014	0.0721
$\alpha_{3\gamma_{2l}\gamma_{3h}}$	Preference for child ability (Type I consumption, Type II child ability)	0.3087	0.0428
$\alpha_{3\gamma_{2h}\gamma_{3l}}$	Preference for child ability (Type II consumption, Type I child ability)	0.2011	0.0230
$\alpha_{3\gamma_{2h}\gamma_{3h}}$	Preference for child ability (Type II consumption, Type II child ability)	0.3083	0.0229
ρ	Weight on future child's ability in the last period	44.2298	11.1550
Panel B. Wage equation			
μ_{ml}	Skill level for Low Type mothers	0.1212	0.0638
μ_{mh}	Skill level for High Type mothers	0.1256	0.0465
μ_1	Coefficient of a mother's years of education	-0.3323	0.0298
μ_2	Coefficient of a mother's age	0.2897	0.0320
μ_3	Coefficient of a mother's race	0.3283	1.0785
μ_4	Coefficient of a mother's cohort	-0.3367	0.0815
μ_5	Coefficient of a mother's macro-area of residence	-0.1283	0.1267
μ_6	Coefficient of a mother's cohort $\times$ macro-area of residence	-0.2356	0.0716
σ_{wage}	Std deviation wage shock	0.4876	0.0276
Panel C. Correlation of labor market skills with preference for child ability)			
$Corr(\mu, \alpha_3)$		-0.0925	

NOTES: In Panel A, Type I corresponds to a low level, and Type II corresponds to a high level of preferences. In Panel B, Low type and High type refer to low and high levels of a mother's unobserved skills in the labor market. Standard errors are estimated with non-parametric bootstrap by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

Table 3
Estimated proportions of types of mothers

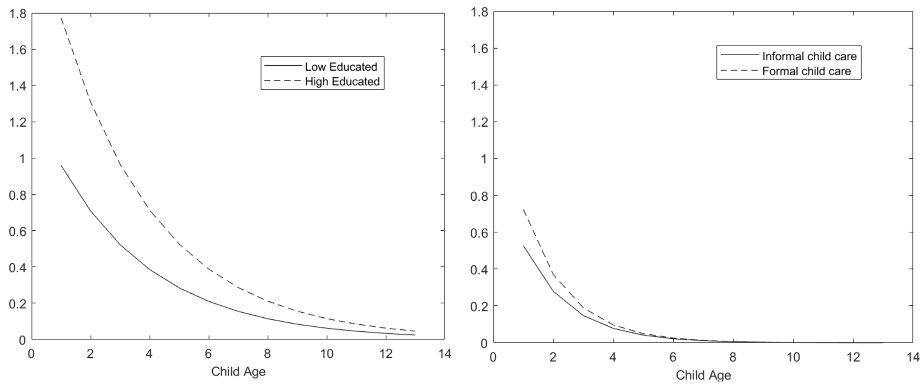
		Estimate	Std. Errors
$\pi_{\gamma_{2l}}$	Proportion Type I consumption	0.5147	0.0764
$\pi_{\gamma_{2h}}$	Proportion Type II consumption	0.4853	(...)
$\pi_{\gamma_{3l}\ \mu_l}$	Proportion Type I child ability & Low Type mothers	0.2197	0.0682
$\pi_{\gamma_{3l}\ \mu_h}$	Proportion Type I child ability & High Type mothers	0.2505	0.0935
$\pi_{\gamma_{3h}\ \mu_l}$	Proportion Type II child ability & Low Type mothers	0.2278	0.0316
$\pi_{\gamma_{3h}\ \mu_h}$	Proportion Type II child ability & High Type mothers	0.3019	0.0571

NOTE. Type I corresponds to a low level, and Type II corresponds to a high level of preferences. Low type and High type refer to low and high levels of a mother's unobserved skills in the labor market. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

Finally, Panel C of Table 2 reports the correlation coefficient between the mother's unobserved skills in the labor market and the preference for her child's ability, which is negative. This suggests that mothers face a trade-off between working and using non-parental child care on the one hand and not working and spending time with their child on the other. The final decisions in terms of time allocation and labor supply depends on the estimated parameters in the CAPF, as well as on the out-of-pocket price of non-parental child care.

Figure 3 reports the time-varying elasticity of a child's cognitive ability with respect to maternal child-care time and to non-parental child care. The figure indicates that the elasticity of a child's cognitive ability with respect to all inputs is higher during the early years and decreases over time, which is in line with previous studies on human capital accumulation (Carneiro and Heckman 2003; Heckman 2008). According to Figure 3-Left, the elasticity of a child's cognitive ability with respect to maternal child-care time varies significantly depending on the mother's level of education. While it ranges between 1.8 at one year of age and 0.1 at age 13 for high-educated mothers, for low-educated mothers it ranges between 1 and 0.1, meaning that the largest differential appears during the child's first years of life. When the child is one year old, a 10% increase in the mother's child care time for high-educated mothers, corresponding to almost 2.5 hours per week, leads to an increase in the level of cognitive ability of the child by 18%. At the same age, for low-educated mothers an increase in their child-care time by 10% leads to an increase in a child's cognitive ability by 10%.

FIGURE 3
Elasticity of a child's cognitive ability with respect to a mother's child-care time and non-parental child care



(A) Mother child-care time (B) Non-parental child care

NOTES: This graph represents the elasticity of a child's cognitive ability with respect to maternal child-care time (τ_t) and non-parental child care (i_t and f_t) as a function of a child's age $t = 1, 2, 3, \ldots 13$. The specification of the parameters is reported in Equations (23), (24), and (25). See Appendix C for further details on the estimation.

Figure 3-Right shows that the elasticity of a child's cognitive ability with respect to non-parental child care does not differ significantly across child-care types, even though formal child care is found to be slightly more productive than informal child care during the child's early years of life. For instance, at one year of age, a 10% increase in formal

child care time corresponding to slightly more than 1 hour per week leads to an increase in the cognitive ability of the child by 7.23%; an increase in informal child care by 10% leads to an increase in a child's ability by 5.26%. This result is in line with the findings in Bernal and Keane (2011) and Loeb et al. (2007) for the US and Hansen and Hawkes (2009) for the UK which state that receiving formal child care before kindergarten age improves a child's language and math competences.

It should be noted that the elasticity of a child's cognitive ability with respect to formal and informal child care start being similar when the child reaches 5 years of age, and it approaches zero from age 9 onward. This pattern could be explained by the different purposes that non-parental child care may have from the mother's point of view once the child starts attending school. In fact, before the child reaches school age, a mother could use non-parental child care not only if she needs someone to look after the child but also if she thinks it can represent an input for the child's subsequent development (which may lead the mother to choose a formal setting). Once the child starts going to school, the educational role of non-parental child care becomes less important, and even more structured environments offering before- or after-school programs may decide to prioritize other activities over educational ones.

The estimated parameters in the CAPF shed light on how the different productivity of inputs affects the mothers' decision-making process, especially their decision to work. Indeed, their final decision on the amount of time to dedicate to the labor market depends on whether the alternative form of care can compensate for the reduction in the mother's child-care time induced by their labor supply. High-educated mothers face a significantly large loss when spending time in the labor market, as the alternative forms of care available –either formal or informal– cannot fully compensate for the reduction in a child's ability induced by their lower child-care time.[46]

A concern for the estimation of the parameters in the CAPF is the absence of other inputs that may be relevant for the child development process. Figure D.1 in Appendix D reports the time-varying estimate of the TFP. Differently from all the other inputs,

[46]This may explain the recent evidence of highly educated women exiting the labor force to care for their children at higher rates than their less educated counterparts. This trend has been reported and analyzed, for instance, by Juhn and Potter (2006) and Macunovich (2010).

the TFP is increasing over time. This seems to suggest that other inputs not explicitly included in the model play a more important role as long as the child ages.[47]

An additional determinant of the mother's decision to use a particular type of child care is represented by its price. Table 4 reports the estimated parameters in the cost equations for informal and formal child care. As previously discussed, the cost determinants in the two equations act as exogenous restrictions for the cost of informal and formal child care; therefore, the coefficients reported in Table 4 allow us to test how well these variables predict the cost of each service. The fact that the coefficients of the state funding for center-based child care and the presence of family members in the neighborhood are both statistically significant is reassuring, as it confirms that in the simulated data these variables represent strong predictors of the child-care costs.

TABLE 4
Estimated parameters in the child care cost equations

		Estimate	Std. Errors
Panel A. Informal child care cost			
λ_{i0}	Intercept	0.2695	0.1166
λ_{i1}	Coefficient of indicator for family members in neighborhood	0.1940	0.0757
$\sigma_{informal\ cost}$	Std deviation informal cost shock	1.0514	0.0145
Panel B. Formal child care cost			
λ_{f0}	Intercept	0.3718	0.1063
λ_{f1}	Coefficient of state funding for center-based child care	0.1434	0.0428
$\sigma_{formal\ cost}$	Std deviation formal cost shock	1.1095	0.0281

NOTES: Standard errors are estimated with non-parametric bootstrap by changing the starting values in each bootstrap iteration. See Appendix C for further details on the estimation.

6.1. ***Fit of the model.*** This section discusses the fit of the model to the data by presenting the actual and simulated moments for the mother's choices and the child's test scores. Table D.3 in Appendix D reports the fit for the other moments.

Table 5 shows the fit of the model for the mother's choice variables, conditional on the age of the child. The overall fit of the model for the mother's choices is good: the model well predicts a larger use of formal child care versus informal child care, and, over time, a negative trend in all investments and a positive trend in labor supply.[48] However, the

[47]In Appendix E, we explicitly consider the role played by a father's time with the child by including this measure in the time investment received by the child at home. The results of this sensitivity analysis suggest that omitting this input in the baseline specification does not affect the estimated elasticity of a child's ability with respect to maternal child-care time but slightly overestimates the elasticity with respect to formal child care.

[48]Although being consistent with the data, the negative trend in the mother's investment decisions may also be due to the assumed specification for the mother's preferences (that is, the mother cares about the child's ability rather than the child's utility) together with the estimated decreasing return on investments over time. This issue may imply that the mother responds to variation in the return on investments and not to variation in the cost of inputs. However, the results of the policy simulations presented in Section

model overestimates the proportion of mothers not working and the mother's child-care time at early ages. This may be due to the assumption about the exogeneity of a mother's wage that is needed in order to solve the model analytically. This assumption implies that the mother does not face any costs associated with her absence from the labor market after childbirth and may thus determine an overestimation of maternal investments in the first periods.

Figure 4 shows the model fit for the child's score measure.[49] The model predicts well the increasing trend in the raw scores over time, even though it underestimates the score in the first periods. The deviation of the simulated test scores from the data in the first years of the child's life may be related to the fact that we observe a measure of cognitive ability from age four onward, and that this is available at early ages only for a limited number of children.

Table 5

Goodness of fit for a mother's choices according to the age of the child

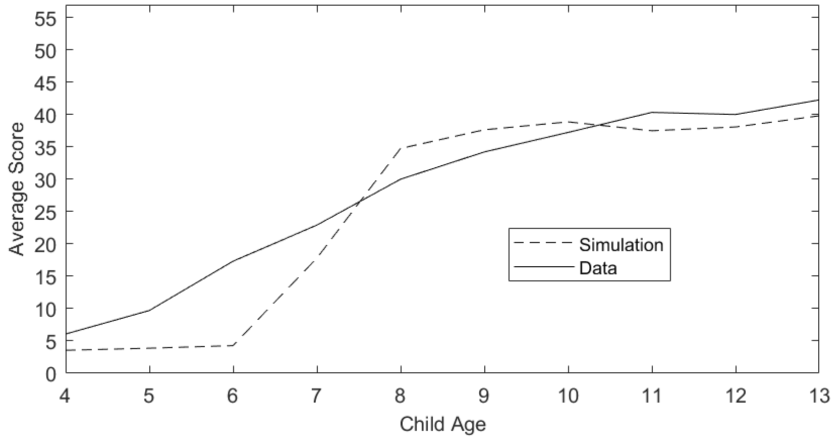
	Child age			
	1-2	3-5	6-10	11-12
Proportion non-working mothers				
Actual data	0.2291	0.2011	0.1780	0.1830
Simulated data	0.4393	0.3301	0.1773	0.1156
Mother's hours of work				
Actual data	24.7488	26.4614	28.0840	29.7518
Simulated data	20.6888	24.2698	36.0740	38.1984
Mother's time with the child				
Actual data	28.5513	29.0493	19.3114	16.3548
Simulated data	50.5177	29.3575	9.3919	17.8782
Informal child care				
Actual data	10.4095	7.7021	2.7569	2.8662
Simulated data	5.9008	3.3511	0.8769	1.5064
Formal child care				
Actual data	12.4029	15.6419	5.2526	4.5742
Simulated data	8.1524	3.6678	0.9340	1.5729

NOTES: Actual data represent PSID-CDS data on children aged 0-12 in 1997 living in intact households without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

7 show that whenever the cost of input is decreased (as is the case for formal and informal child care in all the policies considered), the demand for that input increases.

[49]In Figure 4, the child's test score represents the average between the LW and AP raw scores, and is reported from age four onward because these measures are not available for earlier ages in the data.

Figure 4
Goodness of fit for a child's test scores by age



NOTES: The test score represents the average between the LW and AP test scores in both actual and simulated data. Actual data represent PSID-CDS data on children aged 0-12 in 1997 living in intact households without siblings. See Section 4 and Appendix B for further details on the data. The simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values. The figure reports test scores from age four onward because the test scores measures are not available for earlier ages in the data.

7. Policy simulations

The estimated model predicts two main trade-offs faced by mothers with young children. The first one, which is quite standard in the literature on maternal employment, concerns the choice between spending time at home with the child or being in the labor market and relates to the mother's productivity with the child or at work. The second one, which has not been explicitly addressed in previous studies, refers to the fact that mothers may decide how much time to dedicate to the labor market by accounting for the productivity in the child development process of their own child-care time and of the alternative form of care. The second trade-off is particularly relevant for high-educated mothers, whose child-care time is estimated to be more productive than the alternative non-parental care.

In this section, we use the estimated model to simulate the effects of policies aimed at increasing the mother's opportunities in the labor market or at regulating the child care market. Due to the channels outlined above, the effects of this type of policies are likely to depend on the relative weight that mothers put on their preferences versus the cognitive development of the child. For this reason, and to fully understand the channels described above, we report the results of the policy simulations for the whole sample and separately for high- and low-educated mothers.

7.1. **Wage subsidy policies.** In this subsection, we use the estimated model to simulate the effects of two policies that subsidize mothers' wages. These policies intend to resemble interventions aimed at increasing the participation of mothers in the labor market by lowering labor market taxation or by providing in-work benefits.[50] The first policy assumes that the wage subsidy is the same for all mothers, and it increases the mother's wage offer in every period by 20%. This implies that mothers with a higher wage get a larger increase. The second policy assumes that the wage increase is larger for mothers at the bottom of the wage distribution and is tapered to 0% for mothers at the top. Specifically, the policy simulation is implemented by setting the wage subsidy to 20% for mothers in the first quartile of the wage distribution, to 15% for mothers in the second quartile, to 10% for mothers in the third quartile, and to 0% for mothers in the top quartile of the wage distribution.

TABLE 6
Effects of policies subsidizing the mothers' wages, on the entire sample and by a mother's level of education

	All sample	High educated	Low educated
Panel A. Wage subsidy is the same for all mothers			
Test scores in last period	-0.0110	-0.0216	0.0013
Mother's hours of work	4.0235	5.3113	3.0118
Mother's time with the child	-1.6072	-1.5683	-1.6827
Formal child care	17.1968	13.7376	18.6240
Informal child care	17.2432	13.9188	18.6980
Leisure	-1.5629	-1.6428	-1.4732
Panel B. Wage subsidy larger for low-earning mothers			
Test scores in last period	-0.0073	-0.0193	0.0066
Mother's hours of work	4.0897	5.3086	3.1320
Mother's time with the child	-1.6188	-1.5640	-1.7250
Formal child care	54.1648	30.9729	63.7330
Informal child care	56.4921	32.9215	66.8068
Leisure	-1.5943	-1.6436	-1.5389

NOTES: This table shows the percentage changes with respect to the simulated values of the child's test scores in the last period and the average mothers' choices over the entire period, as induced by the implementation of policies that (i) increase wages by 20% for all mothers (Panel A), and (ii) apply a subsidy ranging between 20% for mothers at the bottom of the wage distribution and 0% for mothers at the top (Panel B).

Table 6 reports the results of the policy simulations. Panel A refers to the 20% subsidy for all mothers (*Policy A*), and Panel B refers to the policy providing a larger subsidy to mothers at the bottom of the wage distribution (*Policy B*). Both policies induce, on average, an increase in the mother's labor supply by 4%, even though such an increase is larger for high-educated mothers than for low-educated mothers. The fact that the labor supply response to both policies is quite similar suggests that mothers at the bottom of the wage distribution are the ones more responsive to the wage change, even when the

[50]Examples of similar policies can be found in various in-work benefit or tax credit reforms introduced, for example, in the UK, where the Child Tax Credit is specifically targeted to households with children.

wage subsidy is the same for all wage levels. Indeed, this group of the population is the one usually targeted by work-enhancing policies. The largest response observed for high-educated mothers is motivated by the fact that, at baseline, this group of mothers face the largest opportunity cost of employment, given by the greater productivity of their child-care time compared to the alternative care providers' time. In the case of policies that make employment more profitable, they show the largest variation in terms of labor supply.

An important implication of these policies is the corresponding increase in the cost of maternal child-care time. As discussed in Section 3, the cost of maternal child-care time depends on the mother's preferences for leisure and on the mother's labor supply. A policy subsidizing the mother's wage determines an increase in labor supply both at the extensive and intensive margins. While Table 6 reports the change in labor supply at the intensive margin, the policies also determine that the proportion of non-working mothers declines from 26% to 22%. Figure D.2 in Appendix D reports the cost of maternal child-care time as a function of a mother's preference for leisure according to a mother's employment status based on the estimated parameters of the model (*Baseline*) and after the simulation of the wage subsidy policy A. The figure shows that with the policy implementation the cost of maternal time for working mothers increases further, while mothers who, after the policy change, still do not work do not face any significant variation in the cost of maternal time.[51]

Finally, the increase in a mother's wage induced by both policies also leads to a greater use of non-parental child care (both formal and informal). In particular, the variation in non-parental child-care use is larger for the low-educated mothers than for the high-educated ones and is greater in the case of Policy B. This suggests that mothers at the bottom of the wage distribution are more responsive to the policy change not only in terms of labor supply but also in terms of formal and informal child-care use.

The final effect on the child's test scores in the last period is very small and differs according to the mother's level of education. Children of high-educated mothers face a negative change in the final test scores, which is induced by the fact that the reduction in their mother's child-care time is not compensated for by the use of (formal or informal)

[51]Policy B induces a variation in the cost of maternal child-care time which is similar to the one of Policy A. Results available upon request.

non-parental child care. Children of low-educated mothers face a slightly positive variation in the final test scores because their mothers' child-care time is less productive and the reduction in their child-care time can be compensated for by the use of non-parental child care.

7.2. **Simulation of policies regulating the child care market.** The model has been estimated using data from the US, where the non-parental child care market is mainly private and heterogeneous in terms of quality and price. The policy maker may be interested in regulating this market by setting rules that guarantee a more homogeneous quality and by providing such services at a subsidized price. To assess the effects of such interventions, we design two types of policies aimed at regulating the non-parental child care market. The first sets the elasticity of a child's ability with respect to informal child care to the level of the elasticity with respect to formal care, according to the estimated values reported in Figure 3-Right by leaving the price unaffected. The second policy also regulates the price by setting the hourly cost for both types of services at a subsidized value of 0.5$ per hour. The first policy implies that the policy maker only changes the quality standards without subsidizing the cost. The second policy instead mimics the case of a subsidized and high-quality child care system where a child care slot in a center-based facility is available to whoever demands it.[52]

Table 7 reports the results of the simulation of the two policies. Panel A reports the results of the policy setting the elasticity of a child's cognitive ability with respect to informal child care to the level of the elasticity with respect to formal child care (*Policy A*), while Panel B reports the results of the policy that, in addition to the regulation, subsidizes non-parental child care (*Policy B*). Policy A implies that informal services are as productive as the formal ones, but less expensive. Therefore, both high- and low-educated mothers switch from the use of formal child care to the use of informal child care. The increase in non-parental child care use leads to a limited increase in labor supply, which also affects the mother's time allocation between child care and leisure. While both high- and low-educated mothers reduce child-care time and leisure, the percentage reduction in child care is lower for high-educated mothers than for their low-educated

[52]This is the context of some Northern European countries, such as Sweden and Norway, where the only type of non-parental child care available is regulated and highly subsidized.

Table 7

Effects of policies regulating the non-parental child care market and subsidizing non-parental child care, on the entire sample and by a mother's level of education

	All sample	High educated	Low educated
Panel A. Both types of child care are regulated			
Test scores in last period	0.0018	0.0000	0.0040
Mother's hours of work	0.9683	0.9806	0.9586
Mother's time with the child	-0.6789	-0.5329	-0.9622
Formal child care	-2.3014	-2.3743	-2.2713
Informal child care	18.9312	20.3184	18.3242
Leisure	-0.2643	-0.1871	-0.3511
Panel B. Both types of child care are regulated and subsidized			
Test scores in last period	0.0349	0.0273	0.0439
Mother's hours of work	0.9683	0.9806	0.9586
Mother's time with the child	-0.6789	-0.5329	-0.9622
Formal child care	63.1795	63.3538	63.1077
Informal child care	94.9248	87.2256	98.2942
Leisure	-0.2643	-0.1871	-0.3511

NOTES: This table shows the percentage changes with respect to the simulated values of the child's test scores in the last period and the average mothers' choices over the entire period, as induced by the implementation of policies that (i) set the elasticity of a child's cognitive ability with respect to informal child care to the level of elasticity with respect to formal child care, according to the estimated values reported in Figure 3-Right (Panel A) and (ii) subsidize both types of non-parental child care services by setting their price to 0.5$/h (Panel B).

counterparts.[53] The final effect on the child's test scores in the last period is very small (i.e., on average it increases by 0.0018%), and differs according to a mother's level of education. Children of high-educated mothers do not have any improvements in their final scores, which suggests that the potential positive effect of receiving a higher-quality care is canceled out by the negative effect induced by the lower time spent with the mother. For children of low-educated mothers, the final effect is positive but extremely small. This suggests that in their case the positive effect of using higher-quality child care slightly dominates the negative effect induced by a lower amount of maternal child-care time.

The main difference between policy A and policy B is that in the context of the latter there is no price discrimination between formal and informal child care, and mothers consider the two types of services as perfect substitutes. The results reported in Panel B of Table 7 show that with policy B the use of both formal and informal child care increases, even though the variation in maternal child-care time and non-parental child care is lower (in absolute value) for high-educated mothers than for low-educated mothers.

[53] The results reported in Table 7 also suggest that policies regulating or subsidizing the non-parental child care market have a modest effect on labor supply and are thus associated with very limited variation in the cost of maternal child-care time for both working and non-working mothers. Evidence of the small effect of child-care subsidy policies on mothers' labor supply is also provided in Rodríguez (2021).

The effect on the child's scores in the last period is larger with policy B than with policy A, and positive for children of both high- and low-educated mothers.

8. Concluding remarks

This paper estimates a model in which labor supply, formal and informal child care, and the time allocation choices of the mother are considered endogenous. The model takes into account that a mother's time productivity can be influenced by her level of education and that non-parental child care may affect the cognitive development of children differently depending on whether it is a formal or informal arrangement. The paper also shows how a mother's labor market participation decision and her child's cognitive ability are affected by the relative productivity of maternal child-care time with respect to the type of non-parental child-care available in the market.

The results show that the elasticity of a child's cognitive ability with respect to maternal child-care time is larger for high-educated mothers than for low-educated mothers, while formal child care is estimated to be more productive than informal child care, especially before the child starts attending primary school. These results may be due to the fact that a child's human capital development positively responds to the cognitive stimulation provided by the caregiver, and this is more likely to occur if the mother is highly educated or the alternative care is offered in a formal setting by trained teachers. However, the results also show that the child-care time of high-educated mothers is the most productive of all inputs.

The simulation of policies that increase the mother's opportunities in the labor market or regulate the non-parental child care market shows that there is a differential effect induced by whether the mother is highly educated or not. While the policies increase the labor supply of all mothers, the high-educated mothers have less incentive to decrease their child-care time than their low-educated counterparts. In fact, for the high-educated mothers, the alternative forms of care cannot fully compensate for the reduction in the child's cognitive ability induced by a lower amount of maternal child-care time. Only the policies enhancing the productivity of non-parental child care determine a non-negative effect on the test scores of all children, while the policies subsidizing mothers' wage have a negative effect on the test scores of children with high-educated mothers.

The analysis leaves space for further research. For instance, little is known about the substitutability or complementarity of a mother's child-care time and non-parental child care in the production of human capital and about whether the production technology would be different for the child's behavioral and non-cognitive development. Future research should better understand how the mother's investment decisions interact in the child's cognitive and non-cognitive development process.

References

Agostinelli, F. and G. Sorrenti (2018). Money vs. time: Family income, maternal labor supply, and child development. *HCEO Working Paper No. 2018-017*. Human Capital and Economic Opportunity Global Working Group.

Becker, G. S. and N. Tomes (1986). Human capital and the rise and fall of families. *Journal of Labor Economics 4*, S1–S39.

Berlinsky, S., M. M. Ferreyra, L. Flabbi, and J. D. Martin (2020). Child care markets, parental labor supply and child development. *IDB Working Paper No. 1163*. Inter-American Development Bank.

Bernal, R. (2008). The effect of maternal employment and child care on children's cognitive development. *International Economic Review 49*(4), 1173–1209.

Bernal, R. and M. P. Keane (2011). Child care choices and children's cognitive achievement: The case of single mothers. *Journal of Labor Economics 29*(3), 459–512.

Bianchi, S. M. (2000). Maternal employment and time with children: dramatic change or surprising continuity? *Demography 37*(4), 401–414.

Blau, D. M. and J. Currie (2006). Pre-school, day care, and after-school care: Who's minding the kids? In E. A. Hanushek and F. Welch (Eds.), *Handbook of The Economics of Education*, Volume 2.

Carneiro, P. and J. J. Heckman (2003). Human capital policy. In J. J. Heckman, A. B. Krueger, and B. M. Friedman (Eds.), *Inequality in America: What Role for Human Capital Policies?*, pp. 77–239. Cambridge: MA: MIT Press.

Caucutt, E., L. Lochner, J. Mullins, and Y. Park (2020). Child skill production: Accounting for parental and market-based time and good investments. *NBER Working Paper No. 27838*. National Bureau of Economic Research.

Cunha, F., J. J. Heckman, L. J. Lochner, and D. V. Masterov (2006). Interpreting the evidence on life cycle skill formation. In E. A. Hanushek and F. Welch (Eds.), *Handbook of the Economics of Education*, pp. 697–812. Amsterdam: North-Holland.

Cunha, F., J. J. Heckman, and S. M. Schennach (2010). Estimating the technology of cognitive and noncognitive skill formation. *Econometrica 78*(3), 883–931.

Del Boca, D., C. J. Flinn, and M. Wiswall (2014). Household choices and child development. *Review of Economic Studies 81*(1), 137–185.

Ermisch, J. and M. Francesconi (2005). Parental employment and children's welfare. In T. Boeri, D. D. Boca, and C. Pissarides (Eds.), *Women at Work: an Economic Perspective*. Oxford University Press.

Ermisch, J. and M. Francesconi (2013). The effect of parental employment on child schooling. *Journal of Applied Econometrics 28*(5), 796–822.

Griffen, A. S. (2019). Evaluating the effects of childcare policies on children's cognitive development and maternal labor supply. *Journal of Human Resources 54*(3), 604–655.

Guryan, J., E. Hurst, and M. Kearney (2008). Parental education and parental time with children. *Journal of Economic Perspectives 22*(3), 23–46.

Hack, M., N. Klein, and H. Taylor (1995). Long-term developmental outcomes of low birth weight infants. *Future Child 5*(1), 176–196.

Hansen, C. and D. Hawkes (2009). Early child care and child development. *Journal of Social Policy 38*(2), 211.

Hart, B. and T. Risley (1995). *Meaningful Differences in the Everyday Experience of Young American Children*. Baltimore: Brookes.

Heckman, J. J. (2007). The economics, technology, and neuroscience of human capability formation. *PNAS 104*(33), 13250–13255. Proceedings of the National Academy of Sciences of the United States of America.

Heckman, J. J. (2008). Schools, skills, and synapses. *Economic Inquiry 46*(3), 289.

Heckman, J. J. and B. Singer (1984). A method for minimizing the impact of distributional assumptions in econometric models for duration data. *Econometrica 52*(2), 271–320.

Hoffert, S. L., P. E. Davis-Kean, J. Davis, and J. Finkelstein (1997). *The Child Development Supplement to the Panel Study of Income Dynamics. The 1997 User Guide*. Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of

Michigan.

ISR (2010a). *The Panel Study of Income Dynamics Child Development Supplement. The User Guide Supplement for CDS-I.* Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of Michigan.

ISR (2010b). *The Panel Study of Income Dynamics Child Development Supplement. The User Guide Supplement for CDS-II.* Ann Arbor, MI: Survey Research Center, Institute for Social Research, The University of Michigan.

Juhn, C. and S. Potter (2006). Changes in labor force participation in the United States. *Journal of Economic Perspectives 20*(3), 27–46.

Loeb, S., M. Bridges, D. Bassok, B. Fuller, and R. W. Rumberger (2007). How much is too much? The influence of preschool centers on children's social and cognitive development. *Economics of Education Review. Economics of Early Childhood Education Issue 26*, 52–66.

Lopez, R. (2003). Are children of young mothers disadvantaged because of their mother's age or family background? *Child Development 74*, 465–474.

Macunovich, D. J. (2010). Reversals in the patterns of women's labor supply in the United States, 1977-2009. *Monthly Labor Review 133*(11), 16–36.

Moschini, E. (2021). Child care subsidies and child skill accumulation in one- and two-parent families. *Working Paper*.

Mroz, T., H. Liu, and W. Van der Klaauw (2010). Maternal employment, migration and child development. *Journal of Econometrics 156*(1), 212–228.

Mullins, J. (2020). A stuctural meta-analysis of welfare reform experiments and their impacts on children. Working paper.

NIEER (2003). *The state of preschool. 2003 State Preschool Yearbook.* National Institute for Early Education Research.

Novick, M. R. (1966). The axioms and principal results of classical test theory. *Journal of Mathematical Psychology 3*, 1–18.

Ribar, D. C. (1992). Child care and the labor supply of married women. Reduced form evidence. *Journal of Human Resources 27*(1), 134–165.

Rodríguez, J. (2021). Understanding the effects of workfare policies on child human capital. *Journal of Labor Economics Forthcoming.*

Todd, P. and K. Wolpin (2003). On the specification and estimation of the production function for cognitive achievement. *The Economic Journal 113*, F3–F33.

US Census Bureau (2000). *Statistical Abstract of the United States: 2000.*

US Census Bureau (2012). *Digest of Education Statistics.* US Department of Commerce.

US Census Bureau (2013). American Time Use Survey 2005-2009. http://www.bls.gov/tus/.

Woodcock, R. W. and M. E. B. Johnson (1989). *Tests of Achievement, Standard Battery [Form B].* Chicago, IL: Riverside Publishing.